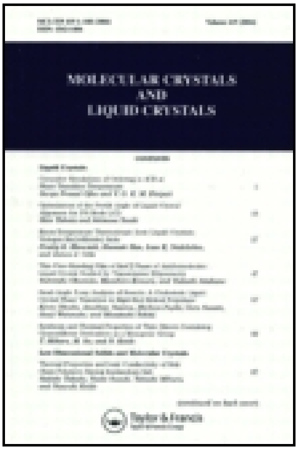

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Deformation Pattern of Twisted Nematic Liquid Crystal Layers in an Electric Field

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At rest between two metallized glass plates a nematic liquid shows a screw-like deformation pattern if one turns the plates by an angle ω_m around an axis perpendicular to the plates. If a voltage U is applied, the molecules are tilted with respect to the electric field when U exceeds a threshold voltage. With increasing amount of twist ω_m the maximum angle of tilt φ_m increases to almost $\pi/2$ even in a weak field. Numerical results are given for the Schadt-Helfrich-cell, i.e. $\omega_m \lambda \pi/2$. The effect of piezo-electricity on deformation in a Schadt-Helfrich-cell is discussed.

INTRODUCTION

The deformation of twisted nematic layers in an electric field has become of considerable interest lately for application in display devices. Schadt and Helfrich[1] have shown how such twisted nematic layers can be switched from light to dark and dark to light, respectively. Leslie[2] has derived the equations which describe the deformation of a twisted orientation pattern in a magnetic field. In the present paper we derive the analogous equations for the deformation of a twisted nematic layer in an electric field. These equations are more complicated due to the dielectric anisotropy and due to the piezo-electric effect.[3,4,5] In the limit of low anisotropy and no piezo-electric effect our equations reduce to those given by Leslie.[2] We present numerical solutions of the equations derived in the present paper.

DEFORMATION OF A TWISTED NEMATIC LAYER WITH NO PIEZO-ELECTRIC EFFECT

Let us assume a nematic liquid crystal layer enclosed between two parallel glass plates which have been given a prefered direction. If one of the plates is rotated by an angle ω_m the nematic layer shows a screw-like orientation pattern. This pattern can be deformed by an electric field perpendicular to the plates if $\epsilon_\parallel > \epsilon_\perp$. Here $\epsilon_\parallel$ and $\epsilon_\perp$ are the dielectric constants of the nematic liquid parallel and perpendicular to the molecular axis, respectively. The resulting orientation pattern is found in the usual way by minimizing the free energy G per unit area of the sample. If the director is denoted by $\vec{n}$ the free energy per unit area in an electric field is given by

$$G = \frac{1}{2} \int_0^L \left\{ k_{11} (\vec{\lambda} \cdot \vec{n})^2 + k_{22} (\vec{n} \cdot \vec{\lambda} \times \vec{n})^2 + k_{33} (\vec{n} \times \vec{\lambda} \times \vec{n})^2 - \vec{D} \cdot \vec{E} \right\} dz \tag{2.1}$$

The vectors $\vec{E}$ and $\vec{D}$ denote the electric field and dielectric displacement, respectively, L denotes the sample thickness. Neglecting any space-charges we have for $\vec{D}$ and $\vec{E}$ the additional equations

$$\vec{\nabla} \times \vec{E} = 0 \qquad \vec{\nabla} \cdot \vec{D} = 0 \tag{2.2}$$

The latter equation states that D_z is a constant. We can then integrate the last term in (2.1) easily to get $-\frac{1}{2} U \cdot D_z$. Let us denote by φ the angle of tilt of $\vec{n}$ with respect to the glass plates and by ω the angle of twist of $\vec{n}$ with respect to the direction of $\vec{n}$ at the boundary $z = 0$. For $z = L$ we have then $\omega = \omega_m$. With this notation we can write (2.1) in the form

$$G = \frac{1}{2} \int_0^L \left\{ (k_{11} \cos^2\varphi + k_{33} \sin^2\varphi) \left(\frac{d\varphi}{dz}\right)^2 + (k_{22} \cos^2\varphi + k_{33} \sin^2\varphi) \cos^2\varphi \left(\frac{d\omega}{dz}\right)^2 \right\} dz - \frac{1}{2} U \cdot D_z \tag{2.3}$$

From the relation

$$D_z = \epsilon_0 \cdot E_z \cdot (\epsilon_\parallel \sin^2\varphi + \epsilon_\perp \cos^2\varphi) \tag{2.4}$$

We get D_z as a functional of $\varphi(z)$[6]

$$D_z = \epsilon_0 \cdot U \Big/ \int_0^L (\epsilon_\parallel \sin^2\varphi + \epsilon_\perp \cos^2\varphi)^{-1} \, dz \tag{2.5}$$

With this expression for D_z we get from (2.3) the Euler-Lagrange equations

$$\frac{d}{dz}\left\{\frac{d\omega}{dz}\left(k_{22}\cos^2\varphi+k_{33}\sin_2^2\varphi\right)\cos^2\varphi\right\}=0 \tag{2.6}$$

$$\frac{d}{dz}\left\{\frac{d\varphi}{dz}\left(k_{11}\cos^2\varphi+k_{33}\sin^2\varphi\right)\right\}=(k_{33}-k_{11})\sin\varphi\cos\varphi\left(\frac{d\varphi}{dz}\right)^2+$$
$$+\left(\frac{d\omega}{dz}\right)^2\left\{(k_{33}-k_{22})\sin\varphi\cos^3\varphi-(k_{22}\cos^2\varphi+k_{33}\sin^2\varphi)\sin\varphi\cos\varphi\right\}-$$
$$-\frac{D_z^2}{\epsilon_o}\frac{(\epsilon_\parallel-\epsilon_\perp)\sin\varphi\cos\varphi}{(\epsilon_\parallel\sin^2\varphi+\epsilon_\perp\cos^2\varphi)^2} \tag{2.7}$$

We can simplify these equations by using the notation

$$\kappa=(k_{33}-k_{11})/k_{11}\qquad \gamma=(\epsilon_\parallel-\epsilon_\perp)/\epsilon_\perp\qquad \alpha=(k_{33}-k_{22})/k_{22} \tag{2.8}$$

with this notation equation (2.6) and (2.7) read

$$\frac{d}{dz}\left\{\frac{d\omega}{dz}\left(1+\alpha\sin^2\varphi\right)\cos^2\varphi\right\}=0 \tag{2.9}$$

$$\frac{d}{dz}\left\{\frac{d\varphi}{dz}\left(1+\alpha\sin^2\varphi\right)\right\}=\kappa\sin\varphi\cos\varphi\left(\frac{d\varphi}{dz}\right)^2+$$
$$+\frac{1+\kappa}{1+\alpha}\left(\frac{d\omega}{dz}\right)^2\left\{\alpha\sin\varphi\cos^3\varphi-(1+\alpha\sin^2\varphi)\sin\varphi\cos\varphi\right\}- \tag{2.10}$$
$$-\frac{D_z^2}{k_{11}\,\epsilon_o\,\epsilon_\perp/\gamma}\cdot\frac{\sin\varphi\cos\varphi}{(1+\gamma\sin^2\varphi)^2}$$

We can integrate equation (2.9) once. For convenience of notation we write the constant of integration in special form

$$\frac{d\omega}{dz}\cdot(1+\alpha\sin^2\varphi)\cos^2\varphi=\frac{D_z\cdot\beta}{\sqrt{k_{11}\,\epsilon_o\,\epsilon_\perp/\gamma}} \tag{2.11}$$

The constant of integration β has to be determined from boundary conditions. We insert the expression (2.11) for $d\omega/dz$ in equation (2.10), multply by $2\omega\phi/dz$ and integrate once.

$$(1+\kappa\sin^2\varphi)\left(\frac{d\varphi}{dz}\right)^2=\frac{D_z^2}{k_{11}\,\epsilon_o\,\epsilon_\perp/\gamma}\quad x$$

$$\times \left\{\gamma^{-1} (1+\gamma \sin^2\varphi)^{-1} - \beta^2 \; [(1+\alpha \sin^2\varphi) \cos^2\varphi]^{-1} - \delta\right\} \tag{2.12}$$

δ is another constant of integration. We can determine δ by the condition, that $\varphi(z)$ has a maximum value φ_m for $z=L/2$. this implies $d\phi/dz=0$ at $z=L/2$. From (2.12) we then get

$$\frac{d\varphi}{dz} = \frac{D_z}{\sqrt{k_{11}\, \epsilon_o\, \epsilon_\perp / \gamma}} \cdot \frac{g(\varphi)}{\sqrt{1+\kappa \sin^2\varphi}} \tag{2.13}$$

where we have introduced a function

$$g(\varphi) = \left\{ \frac{\sin^2\varphi_m - \sin^2\varphi}{(1+\gamma \sin^2\varphi)\cdot(1+\gamma \sin^2\varphi_m)} - \beta^2 \frac{1+\kappa}{1+\alpha} \left[(1+\alpha \sin^2\varphi_m)\cos^2\varphi\right]^{-1} + \beta^2 \frac{1+\kappa}{1+\alpha} \left[(1+\alpha \sin^2\varphi_m)\cos^2\varphi_m\right]^{-1} \right\}^{1/2} \tag{2.14}$$

From (2.11) and (2.13) we can calculate $\omega(z)$

$$\omega(z) = \beta. \int_0^{\varphi(z)} \frac{\sqrt{1+\alpha \sin^2\varphi}}{g(\varphi)\cos^2\varphi\,(1+\alpha \sin^2\varphi)} \, d\varphi \tag{2.15}$$

For $z=L/2$ we have $\varphi=\varphi_m$ and $\omega=\omega_m/2$. This yields the equation

$$\omega_m = f_1(\varphi_m, \beta) = 2\cdot\beta \int_0^{\varphi_m} \frac{\sqrt{1+\kappa \sin^2\varphi}}{g(\varphi)\cos^2\varphi\,(1+\alpha \sin^2\varphi)} \, d\varphi \tag{2.16}$$

From (2.5) and (2.13) we get

$$U/U_o = \frac{2}{\pi} \int_0^{\varphi_m} \frac{\sqrt{1+\kappa \sin^2\varphi}}{(1+\gamma \sin^2\varphi)\, g(\varphi)} \, d\varphi = f_2(\varphi_m, \beta) \tag{2.17}$$

Where we have defined a natural unit of voltage

$$U_o = \pi \left(\frac{k_{11}}{\epsilon_o \cdot (\epsilon_\parallel - \epsilon_\perp)}\right)^{1/2} \tag{2.18}$$

Integrating the inverse of equation (2.13) leads to

$$z = \frac{\sqrt{k_{11}\, \epsilon_o\, \epsilon_\perp / \gamma}}{D_z} \cdot \int_0^{\varphi(z)} \frac{\sqrt{1+\kappa \sin^2\varphi}}{g(\varphi)} \, d\varphi \tag{2.19}$$

For z =L/2 we have $\varphi=\varphi_m$. From this condition we can determine the prefactor in (2.19)

$$z/L = \frac{1}{2} \int_0^{\varphi} \frac{\sqrt{1+\kappa \sin^2\varphi}}{g(\varphi)} d\varphi \bigg/ \int_0^{\varphi_m} \frac{\sqrt{1+\kappa \sin^2\varphi}}{g(\varphi)} d\varphi \tag{2.20}$$

For any given values of U and ω_m we have to solve the non-linear equations (2.16) and (2.17) to determine the parameters φ_m and β. Then we can find the angle of tilt φ as a function of z from (2.20). With the aid of equation (2.15) we can then find the angle of twist ω as a function of z. We can solve the equations (2.16) and (2.17) in the limit $\varphi_m \to 0$. In this limit we find

$$g(\varphi) \approx \sqrt{1+\beta^2 \frac{1+\kappa}{1+\beta^2}(1-\alpha)} \cdot \sqrt{\varphi_m^2 - \varphi^2}$$

with this expression for φ equation (2.16) and (2.17) reduce to

$$\omega_m = \pi \cdot \beta \cdot \left\{1+\beta^2 \frac{1+\kappa}{1+\alpha}(1-\alpha)\right\}^{-1/2} \tag{2.21}$$

$$U/U_0 = \left\{1+\beta^2 \frac{1+\kappa}{1+\alpha}(1-\alpha)\right\}^{-1/2} \tag{2.22}$$

Equation (2.22) gives the limiting value for U for which φ_m goes to zero. This is the threshold-voltage which we denote by U_T. Below U_T there is no deformation. For $\omega_m=0$ we have $\beta=0$ from (2.21) and therefore $U_T = U_0$. We can solve (2.21) for β and insert for α and κ

$$\beta = \left\{\left(\frac{\pi}{\omega_m}\right)^2 + \frac{k_{33}}{k_{11}} - 2\frac{k_{22}}{k_{11}}\right\}^{-1/2} \tag{2.23}$$

Inserting in (2.22) yields the threshold-voltage

$$U_T = U_0 \cdot \left\{1 + \left(\frac{\omega_m}{\pi}\right)^2 \cdot \left(\frac{k_{33}}{k_{11}} - 2\frac{k_{22}}{k_{11}}\right)\right\}^{1/2} \tag{2.24}$$

The equivalent expression for the critical magnetic field has been given by Leslie.[2] From Leslie's result Schadt and Helfrich[1] conjectured expression (2.24) for the critical voltage. For small dielectric anisotropy the difference $\epsilon_\parallel - \epsilon_\perp$ is important only for the threshold voltage. We can then keep U_T fixed and take the limit $\gamma \to 0$ in the expression (2.16) and (2.17) for f_1 and f_2. We then get exactly Leslie's result if we only replace the voltage U by the magnetic field H and the threshold voltage U_T by the threshold field H_T.

THE EFFECT OF PIEZO-ELECTRICITY ON THE DEFORMATION OF A TWISTED NEMATIC LAYER

As pointed out first by R. B. Meyer a nematic liquid may show an electric polarisation $\vec{P}$ in reponse to mechanical deformation. In real display devices this piezo-electric polarisation will be screened out by space charges. In very pure samples, however, the piezo-electric polarisation might influence the deformation pattern slightly. For completness we therefore give a mathematical treatment of this effect in this chapter. Adopting the notation of Meyer we have for $\vec{P}$

$$\vec{P} = e_{11}\, \vec{n} \cdot \mathrm{div}\, \vec{n} - e_{33}\, \vec{n} \times \mathrm{rot}\, \vec{n} \tag{3.1}$$

Instead of equation (2.4) we now have a more complicated relation between $\vec{D}$ and $\vec{E}$

$$D_z = \epsilon_o E_z (\epsilon_\parallel \sin^2\varphi + \epsilon_\perp \cos^2\varphi) + (e_{11} + e_{33}) \cos\varphi \sin\varphi \frac{d\varphi}{dz} \tag{3.2}$$

It should be noted that the relation (2,5) between D_z and the voltage U is still valid. The free energy G has to be changed by a term[5]

$$-\frac{1}{2}\int_o^L P_z \cdot E_z\, dz = \frac{1}{2}\, \frac{(e_{11} + e_{33})^2}{\epsilon_o} \int_o^L \frac{\cos^2\varphi \sin^2\varphi}{(\epsilon_\parallel \sin^2\varphi + \epsilon_\perp \cos^2\varphi)} \left(\frac{d\varphi}{dz}\right)^2 dz \tag{3.3}$$

Adding this term to the expression (2.3) we get for the free energy

$$G = \frac{k_{11}}{2}\int_o^L \left\{\left(1 + \kappa \sin^2\varphi + \Delta \frac{\cos^2\varphi \sin^2\varphi}{1+\gamma \sin^2\varphi}\right)\left(\frac{d\varphi}{dz}\right)^2 + \right.$$
$$\left. + \frac{1+\kappa}{1+\alpha}\,(1+\alpha \sin^2\varphi)\cos^2\varphi \left(\frac{d\omega}{dz}\right)^2\right\} dz - \frac{1}{2}\, U \cdot D_z \tag{3.4}$$

The parameter Δ in this expression is given by

$$\Delta = (e_{11} + e_{33})^2 / (k_{11}\, \epsilon_o\, \epsilon_\perp)$$

The calculation now proceeds exactly as in ch. 2 to give

$$f_1(\varphi_m, \beta) = 2\beta \int_o^{\varphi_m} \frac{\sqrt{1+\kappa \sin^2\varphi + \Delta \cos^2\varphi \sin^2\varphi / (1+\gamma \sin^2\varphi)}}{g(\varphi) \cos^2\varphi\, (1+\alpha \sin^2\varphi)}\, d\varphi \tag{3.5}$$

$$f_2(\varphi_m, \beta) = \frac{2}{\pi}\int_o^{\varphi_m} \frac{\sqrt{1+\kappa \sin^2\varphi + \Delta \cos^2\varphi \sin^2\varphi / (1+\gamma \sin^2\varphi)}}{g(\varphi)\, (1+\gamma \sin^2\varphi)}\, d\varphi \tag{3.6}$$

and

$$\frac{z}{L}=\frac{1}{2}\,\frac{\int_{0}^{\varphi}\sqrt{1+\kappa\sin^2\varphi+\Delta\cos^2\varphi\sin^2\varphi/(1+\gamma\sin^2\varphi)}\;/\,g(\varphi)\,d\varphi}{\int_{0}^{\varphi_m}\sqrt{1+\kappa\sin^2\varphi+\Delta\cos^2\varphi\sin^2\varphi/(1+\gamma\sin^2\varphi)}\;/\,g(\varphi)\,d\varphi} \tag{3.7}$$

These expressions reduce to equations (2.16), (2.17) and (2.20) in the limit $\Delta\to 0$.

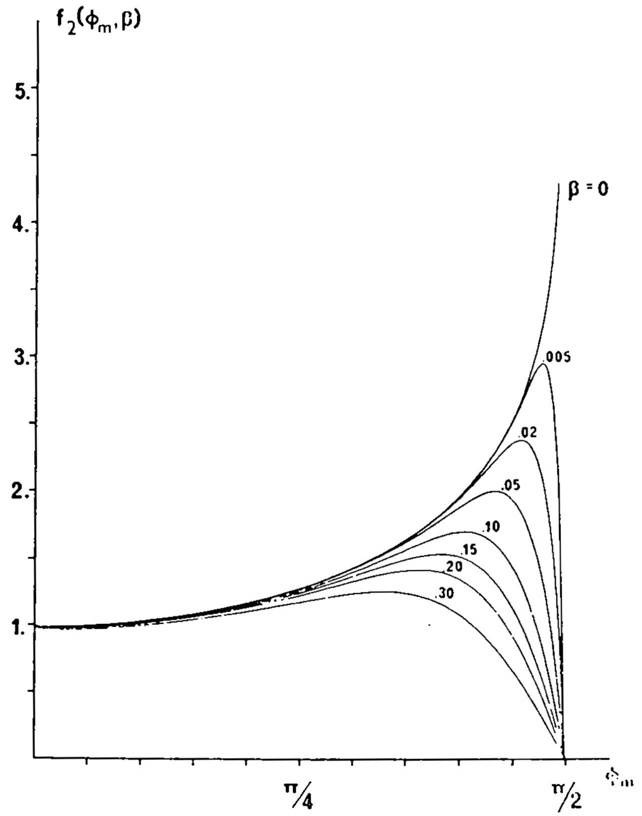

FIGURE 1 Function $f_2(\varphi_m,\beta)$ versus φ_m for $\alpha=.6$, $\chi=.8$, $\gamma=o$ and various values of β.

NUMERICAL RESULTS AND DISCUSSION

For given values of U and ω_m we have to solve the two non-linear equations (2.16) and (2.17) for the parameters φ_m and β. We first plot the function $f_2(\varphi_m, \beta)$ versus φ_m for various values of β. Such a plot is shown in figure 1 for the parameters α=.6, κ=.8 and γ=0. For β=0 the curve f_2 versus φ_m starts from 1 and goes to infinity as φ_m approaches $\pi/2$. For $\beta \neq 0$ the curve starts from a value given by expression (2.22), assumes a maximum value and goes to zero as φ_m approaches $\pi/2$. We can use this plot to find the solutions of equation (2.17). If U/U_0 is less than the limiting value given by equation (2.22) we have no solution for β=0 and one solution for $\beta \neq 0$. We can rule out this solution as unphysical, however, because it goes to $\pi/2$ as U goes to zero. If U/U_0 is greater than the threshold value (2.22) there is one solution of equation (2.17) for β=0. If $\beta \neq 0$ but not too large, there are two roots of equation (2.17). If β increases the first root increases and the second decreases til they coincide for a limiting value of β. For values of β greater than this no solution of equation (2.17) exists. For given value of U/U_0 we find the roots of equation (2,17) for every value of β. Inserting these roots into equation (2.16) we get a function $f_1(\beta)$ for every given value of U/U_0. Figure 2 shows a plot of this function $f_1(\beta)$. For every β we have two values of f_1 corresponding to the two solutions of equation (2.17). With increas-

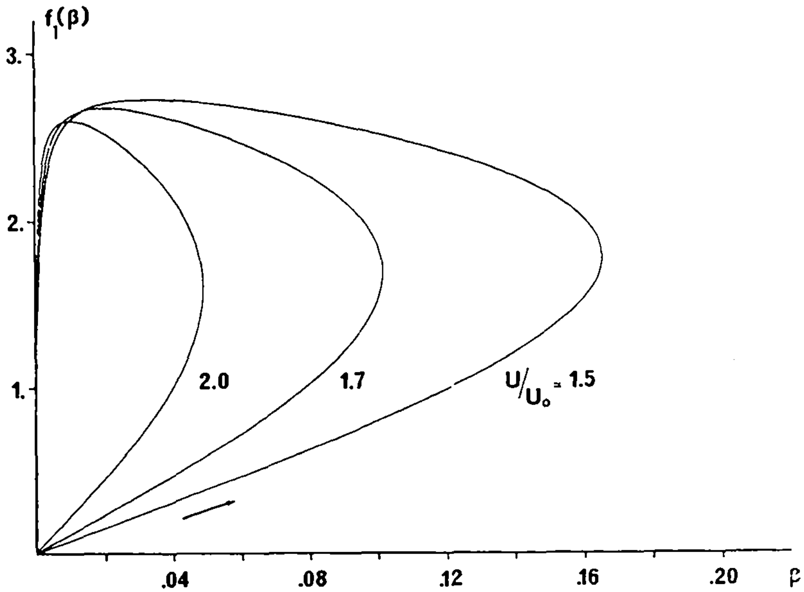

FIGURE 2 Function $f_1(\beta)$ versus β for α=.6, α=.8, γ=o and various values of U/U_0.

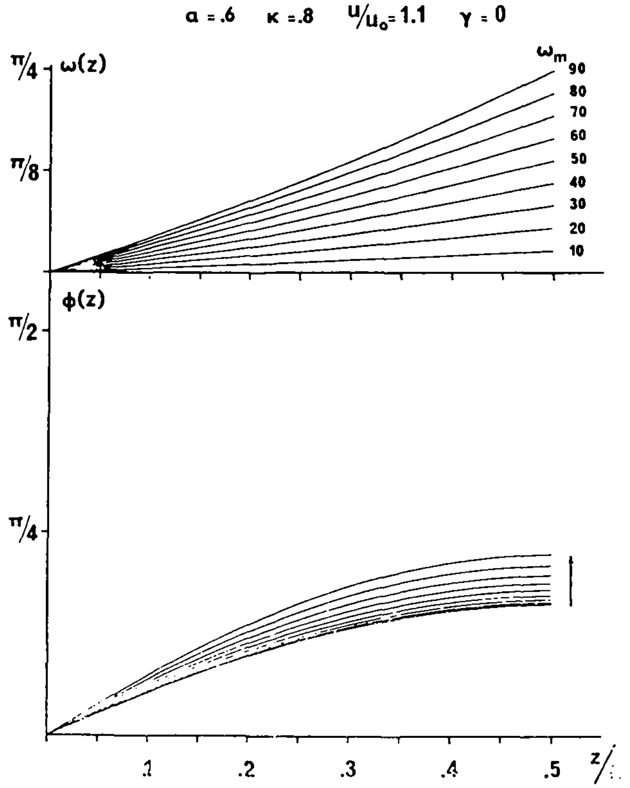

FIGURE 3 The angle of tilt $\varphi(z)$ and the angle of twist $\omega(z)$ versus z/L for various values of ω_m.

ing β we reach the limiting value of β for which the two solutions of (2.17) coincide (see fig. 1) and at this point therefore the two branches of $f_1(\beta)$ meet. For even larger values of β the function $f_1(\beta)$ is not defined. From figure 2 we see that for a given value of U/U_0 we find two solutions of equation (2,16) for every value of ω_m. We will see below that we always have to choose the larger value for β of these two roots of equations (2.16). We now start from a parallel orientation with ω_m=o and U=o. We apply a voltage U/U_0=1.5. For ω_m = o we have β=o and find the solution of equation (2.17) from figure 1. Now we twist the sample by a small amount ω_m. Now β is greater than zero and therefore we have two roots φ_m for equation (2.17) (see fig. 1). For reasons of continuity we have to choose the lower value of φ_m. This corresponds to the lower branch of $f_1(\beta)$ shown in figure 2. So we see that with increasing ω_m we have to follow the arrow along the lower branch of $f_1(\beta)$ in figure 2. This is to say that we have to choose the larger value for β of the two

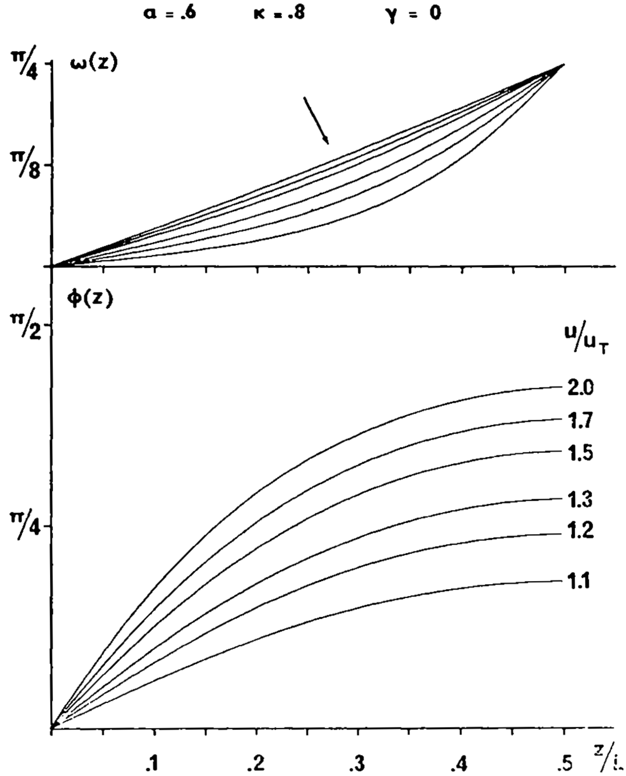

FIGURE 4 $\varphi(z)$ and $\omega(z)$ versus z/L for various values of U/U_T. The arrow indicates the direction of increasing U.

possible roots of equation (2.16). With increasing values of ω_m β also increases til it reaches a maximum value. For this value of β the two branches of $f_1(\beta)$ are connected and the two roots of equation (2.17) coincide (see fig 1). If we increase ω_m further we have to proceed along the upper branch of $f_1(\beta)$ in figure 2. With increasing ω_m β decreases. The maximum angle of tilt φ_m however increases further as we have to choose the larger of the two roots of equation (2.17) now being on the upper branch of $f_1(\beta)$. From figure 2 we see that $f_1(\beta)$ and therefore ω_m has a maximum value. At his point β is fairly small and φ_m therefore is almost $\pi/2$ as is seen from figure 1. If we twist the sample further it has to switch discontinuously to another type of solution of equations (2.9) and (2.10) which is governed by different boundary conditions. This will be discussed in the appendix. So far we have discussed the solution of the non-linear equations (2.16) and (2.17). Having found the parameters φ_m and β for any given values of U/U_0 and ω_m we can use equations (2.20) and (2.15) to calculate the angle of tilt $\varphi(z)$ and the angle of twist $\omega(z)$ as functions of

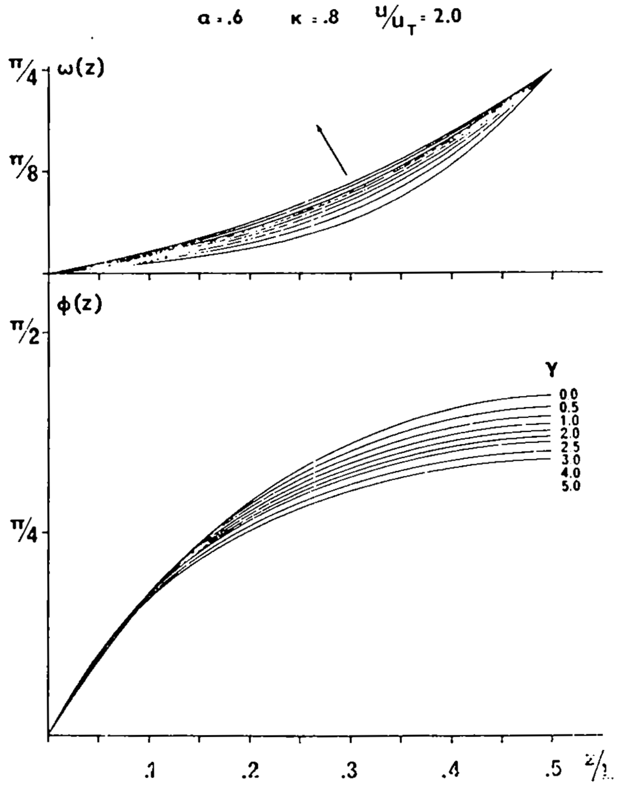

FIGURE 5 $\varphi(z)$ and $\omega(z)$ versus z/L for various of γ. The arrow indicates the direction of increasing γ.

coordinate z. The result is shown in figure 3 for $U/U_o = 1.1$ and ω_m varying from 10 to 90 degrees. From this plot we see how the angle of tilt increases with increasing amount of twist. U_o has been chosen as unit voltage in this figure rather than U_T because U_T depends on the amount of twist ω_m. Figure 4 shows similar curves for the Schadt-Helfrich-cell, that is $\omega_m = \pi/2$. The voltage is given in units of threshold voltage U_T rather than U_o. Figure 5 shows similar curves for constant voltage but different values of dielectric anisotropy γ ranging from o to 5. Figure 6 finally shows the maximum angle of tilt φ_m as a function of U/U_T for various values of γ.

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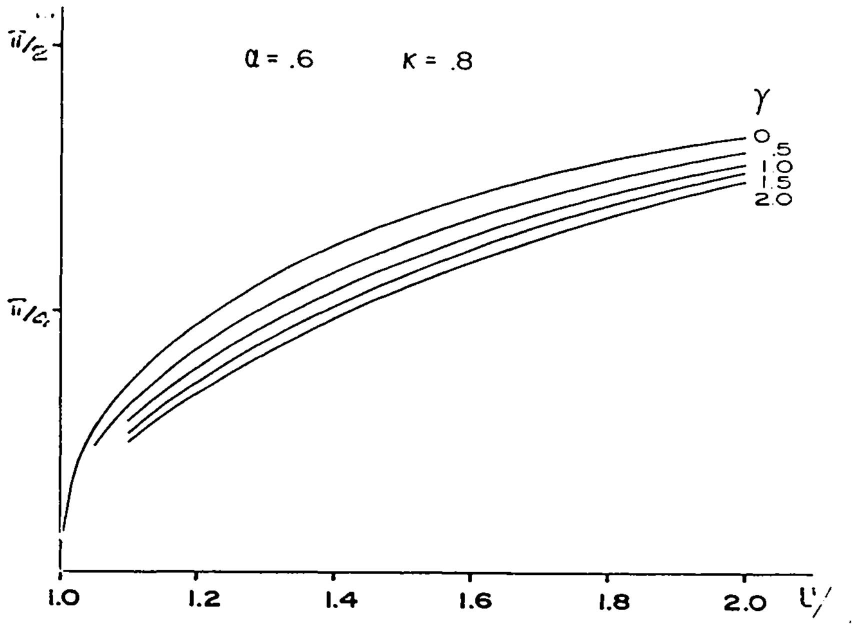

FIGURE 6 φ_m versus U/U_T for various values of γ.

Appendix

If we apply a voltage above threshold to a parallel oriented sample and then twist the sample gradually to $\omega_m=\pi$ we expect to end up with $\varphi=o$ at $z=o$, $\varphi=\pi/2$ at $z = L/2$ and $\varphi=\pi$ at $z=L$. In ch 4 we have seen that we can never reach the situation $\varphi_m = \pi/2$ and $\omega_m=\pi$ with the solution given in ch. 2. Another type of solution must exist which will be discussed below. For given values of U and ω_m we can generate the corresponding orientation pattern in two ways. We can start from a parallel orientation, apply the voltage and twist counter clock wise by an angle ω_m. In this case our boundary condition is that $\varphi(z)$ assumes a maximum value φ_m at $z=L/2$. We used this condition in ch. 2 to determine the constant of integration δ. But we can also start from a metastable case with $\varphi=0$ at $z=o$, $\varphi=\pi/2$ at $z=L/2$ and $\varphi=\pi$ at $z = L$, apply the voltage and now twist clock wise by an angle $\pi-\omega_m$. In this case we have to use the boundary condition $\varphi=\pi/2$ at $z = L/2$ to determine δ. Rewriting equation (2.12) we have

$$\frac{d\varphi}{dz} = \frac{D_z}{\sqrt{k_{11}\ \epsilon_o\ \epsilon_\perp/\gamma}} \cdot \frac{k\,(\varphi,\delta)}{\sqrt{1+\kappa \sin^2\varphi}} \qquad \text{(A 1)}$$

with

$$k(\varphi, \delta) = \left\{\gamma^{-1} (1 + \gamma \sin^2\varphi)^{-1} - \beta^2 \frac{1+\kappa}{1+\alpha} [(1+\alpha \sin^2\varphi) \cos^2\varphi]^{-1} - \delta\right\}^{1/2} \tag{A 2}$$

We can now proceed exactly as in ch. 2 to get the equations

$$\pi - \omega_m = 2\beta \int_0^{\pi/2} \frac{\sqrt{1+\kappa \sin^2\varphi}}{1+\alpha \sin^2\varphi} \cdot \frac{d\varphi}{k(\varphi, \delta)} \tag{A 3}$$

$$U/U_o = \frac{2}{\pi} \int_0^{\pi/2} \frac{\sqrt{1+\kappa \sin^2\varphi}}{1+\gamma \sin^2\varphi} \cdot \frac{d\varphi}{k(\varphi, \delta)} \tag{A 4}$$

We can solve these equations numerically for β and δ in much the same way as we solved equations (2.16) and (2.17) for β and φ_m. If we vary ω_m between $\pi/2$ and π we may expect a hysteresis to occur as the sample switches between the two types of solution given by equations (2.16) (2.17) and (A 3) (A 4), respectively.

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